FINITE-AMPLITUDE THERMOCAPILLARY CONVECTION IN A TWO-LAYER SYSTEM

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Thermocapillary motion in a system of actual liquids is investigated on the basis of nonlinear equations. The existence of both steady-state and oscillatory convection conditions has been established.

Linear problems of convective equilibrium stability of two-layer systems were investigated in [1-4]. An investigtion of finite-amplitude convective motion in a system with an interface, produced by the thermocapillary mechanism, was performed in [5]. Systems of liquids with simulator parameters were considered in that paper.

1. Assume that a cavity with a rectangular cross section is filled with two viscous, immiscible liquids with different characteristics. The coordinate origin is placed at the interface, the x axis is horizontal, and the y axis is directed vertically upward. The interface is assumed to be flat and nondeformable. The case of equal-thickness layers is contemplated. The surface tension coefficient is a linear function of the temperature, $\sigma = \sigma_0 - \alpha T$. The convection caused by buoyancy is neglected.

All the quantities pertaining to the liquid filling the $0 < x < \ell$, $0 < y < a_1$ region are deoted by subscript 1, while the quantities pertaining to the liquid filling the $0 < x < \ell$, $-a_2 < y < 0$ region are denoted by subscript 2. The coefficients of dynamic and kinematic viscosity, thermal conductivity, and thermal diffusivity are denoted by η_i, ν_i, κ_i and χ_i ($i = 1, 2$). We shall subsequently use the following notation: $\eta = \eta_1/\eta_2$, $\nu = \nu_1/\nu_2$, $\varkappa = \varkappa_1/\varkappa_2$, $\chi = \chi_1/\chi_2$, $L = l/a_1$, and $a = a_2/a_1$. We shall use a_1, a_1^2/ν_1, ν_1, ν_1/a_1^2 and θ as the units of length, time, stream function, vorticity, and temperature, respectively (see Notation). Then, the complete nonlinear equations describing two-dimensional convection in both layers are written thus:

$$\frac{\partial\varphi_i}{\partial t} + \frac{\partial\psi_i}{\partial y}\frac{\partial\varphi_i}{\partial x} - \frac{\partial\psi_i}{\partial x}\frac{\partial\varphi_i}{\partial y} = b_i\Delta\varphi_i,$$
$$\Delta\psi_i = -\varphi_i, \tag{1}$$
$$\frac{\partial T_i}{\partial t} + \frac{\partial\psi_i}{\partial y}\frac{\partial T_i}{\partial x} - \frac{\partial\psi_i}{\partial x}\frac{\partial T_i}{\partial y} = \frac{c_i}{P}\Delta T_i.$$

Here, $b_1 = c_1 = 1$, $b_2 = 1/\nu$, and $c_2 = 1/\chi$.

The boundary conditions at the horizontal solid walls are given by

$$y = 1,\ \psi_1 = \frac{\partial\psi_1}{\partial y} = 0;\ T_1 = 1;\quad y = -a,\ \psi_2 = \frac{\partial\psi_2}{\partial y} = 0;\ T_2 = 0. \tag{2}$$

Two types of lateral walls are considered:

a) free walls,

$$x = 0, L,\ \psi_i = 0,\ \frac{\partial^2\psi_i}{\partial x^2} = 0,\ \frac{\partial T_i}{\partial x} = 0, \tag{3}$$

b) solid walls,

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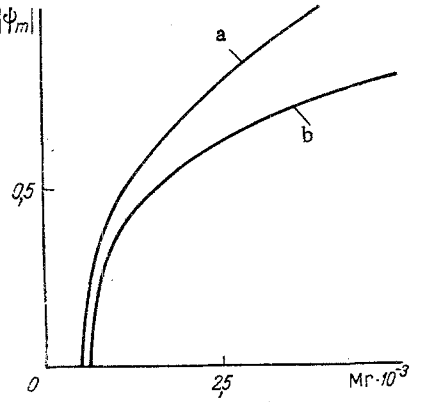

Fig. 1

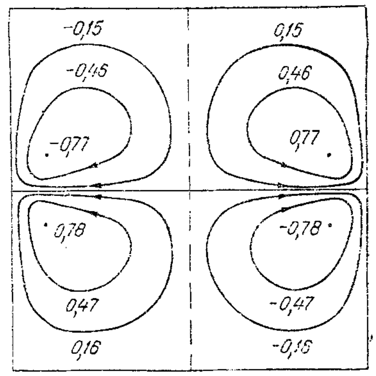

Fig. 2

Fig. 1. Absolute maximum value of the stream function $|\psi_m|$ as a function of the Mr number (a, free lateral boundaries; b, solid lateral boundaries).

Fig. 2. Stream function patterns for the steady-state motion.

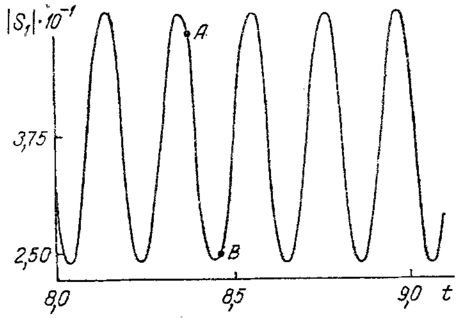

Fig. 3. Dependence of $|S_1|$ on t (Mr = 5000).

$$x = 0,\ L,\ \psi_i = \frac{\partial\psi_i}{\partial x} = 0,\ T_1 = \frac{y + \varkappa a}{1 + \varkappa a},\ T_2 = \frac{\varkappa(y + a)}{1 + \varkappa a}. \tag{4}$$

The following conditions are satisfied at the interface:

$$\begin{gathered} y = 0,\ \psi_1 = \psi_2 = 0,\ \frac{\partial\psi_1}{\partial y} = \frac{\partial\psi_2}{\partial y},\ T_1 = T_2, \\ \varkappa\frac{\partial T_1}{\partial y} = \frac{\partial T_2}{\partial y},\quad \varphi_2 = \eta\varphi_1 + \mathrm{Mr}\frac{\partial T_1}{\partial x}. \end{gathered} \tag{5}$$

The boundary problem (1)-(5) comprises dimensionless numbers — the Marangoni number M (or the related parameter Mr), the Prandtl number P, the ratios of the parameters of the first and the second liquid defined above, η, ν, κ, and χ, and the geometric parameters L and a:

$$M = \frac{\alpha\theta a_1}{\eta_1\chi_1},\ \mathrm{Mr} = \frac{\eta M}{P},\ P = \frac{\nu_1}{\chi_1}.$$

2. The problem was solved by means of finite differences. We used the explicit scheme of the determination method with central differences. The basic calculations were performed with a 16 × 32 grid, while the control calculations were performed with a 20 × 40 grid. The Poisson equation was solved by using the iteration method. The iteration accuracy for the Poisson equation was equal to 10^{-4} in calculating the steady-state motion, and 10^{-8} in calculating the oscillatory motion. The Thom equation was used for approximating the vorticity

at the solid boundaries [6]. At the interface, the vorticity in the upper-lying liquid was calculated by means of the expression

$$\varphi_1(x,\ 0) = \frac{-2[\psi_2(x,\ -\Delta y) + \psi_1(x,\ \Delta y)]}{(\Delta y)^2(1+\eta)} - \mathrm{Mr}\,\frac{1}{1+\eta}\,\frac{\partial T_1}{\partial x}(x,\ 0),$$

the derivation of which is similar to that of Thom's equation; the vorticity in the lower-lying liquid was calculated by using the last of Eqs. (5). Here, Δy is the grid spacing along the vertical coordinate. The temperature at the interface was determined by means of the relationship

$$T_1(x,\ 0) = T_2(x,\ 0) = \frac{[4T_2(x,\ -\Delta y) - T_2(x,\ -2\Delta y)] + \varkappa[4T_1(x,\ \Delta y) - T_1(x,\ 2\Delta y)]}{3(1+\varkappa)}.$$

The time interval was chosen on the basis of calculation stability.

3. Consider the water—oil system of liquids (Dow Corning N 200) with the following characteristics: $\eta = 0.915$, $\nu = 1.116$, $\kappa = 0.169$, $\chi = 0.472$, P = 6.28, L = 2 and a = 1.

If the temperature gradient is perpendicular to the interface, mechanical equilibrium is possible in the system. If the threshold value of the Mr number is exceeded, the equilibrium becomes unstable, and thermocapillary convection develops in the system (Fig. 1). Convective motion develops simultaneously in both layers. Since the coefficients of dynamic and kinematic viscosity of the liquids are close to each other, the curves of maximum motion intensity in the first and the second liquids virtually coincide within the scale of the diagram.

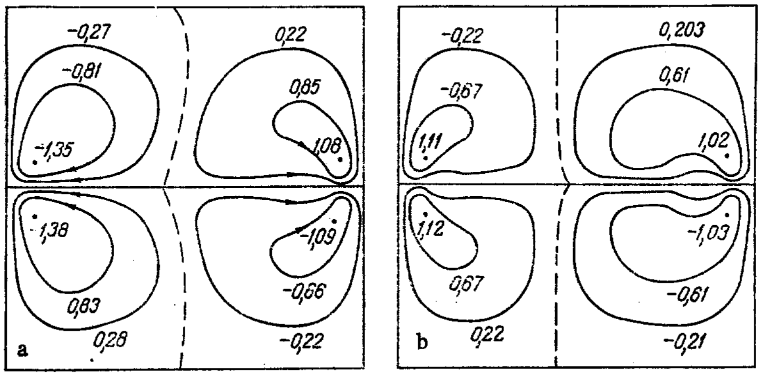

Fig. 4. Stream function patterns corresponding to points A (a) and B (b) of Fig. 3.

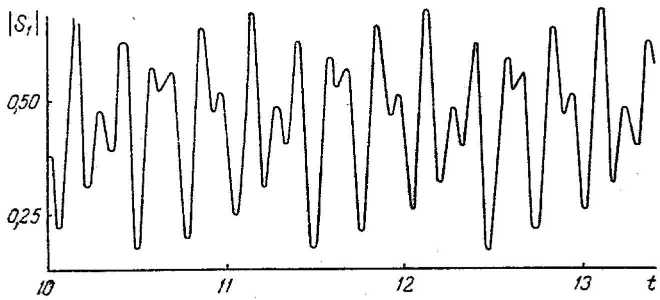

Fig. 5. Dependence of $|S_1|$ on t (Mr = 7500).

Let us examine the case of free lateral boundaries.

It is evident from the pattern of the stream function isolines (Fig. 2) that the flow velocity is at a maximum near the interface, while the vortices are flattened, as it were, against the interface.

We introduce the quantity

$$S_1 = \int_0^1 dy \int_0^{L/2} \psi_1(x, y, t)\, dx$$

as a characteristic of the flow structure. Comparison between the stream function isolines indicates that the flow structure changes insignificantly during the oscillatory process (Figs. 3 and 4). A further increase in the Mr number is accompanied by oscillations of increasing complexity (Fig. 5), which maintain a regular character in the investigated range of the Mr parameter (up to Mr = 9000).

The convection threshold rises for a cavity where all boundaries are solid (see Fig. 1). The steady-state flow structure is similar to that shown in Fig. 2.

NOTATION

x, y, Cartesian coordinates; η, dynamic viscosity coefficient; ν, kinematic viscosity coefficient; κ, thermal conductivity coefficient; χ, thermal diffusivity coefficient; ψ, stream function; ϕ, vorticity; T, temperature; θ, temperature difference between the horizontal boundaries; L and a, geometric parameters; σ, surface tension coefficient; M, Marangoni number; P, Prandtl number. Subscripts: 1, quantities pertaining to the higher-lying liquid; 2, quantities pertaining to the lower-lying liquid.

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